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Dynamics of Predator-Prey Model with Holling Type II Involving Predator Stage-Structured and Cannibalism

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Abstract

Predator–prey models with nonlinear functional responses provide a robust framework for understanding population regulation and oscillatory dynamics. This study analyzes a three-dimensional predator–prey model incorporating Holling type II functional responses, explicit predator stage structure, and cannibalism. The primary objective is to investigate how adult predation, maturation, and cannibalism parameters influence equilibrium stability and the emergence of oscillations through Hopf bifurcation.

Analytical results establish the existence and local stability of a positive coexistence equilibrium. For a biologically relevant parameter set, numerical simulations demonstrate that trajectories converge to the coexistence equilibrium (59.9078, 23.9092, 56.0552). This state is locally asymptotically stable, as the real parts of all Jacobian eigenvalues are negative. Numerical continuation methods are employed to detect Hopf bifurcations induced by key parameters. Two Hopf points are identified for the adult predation rate at 0.4211 and 8.7725, while the maturation rate induces bifurcations at 0.2661 and 0.5271. Additionally, the cannibalism parameter triggers a Hopf bifurcation at 0.1835, initiating periodic population oscillations. These results demonstrate that maturation and cannibalism define distinct instability thresholds, jointly governing the transition from stable coexistence to sustained oscillatory dynamics in stage-structured systems.

Keywords: Cannibalism; Dynamical Analysis; Holling Type II; Hopf Bifurcation; Stage-Structured

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1 Introduction

Predator–prey interactions constitute a fundamental topic in theoretical ecology, as they govern population regulation, species coexistence, and ecosystem stability. Mathematical models provide a fundamental framework for understanding these interactions by revealing how nonlinear feedback mechanisms shape population persistence, extinction, and oscillatory dynamics [1], [2]. Over the past decades, predator–prey modeling has evolved continuously to incorporate biologically relevant mechanisms that improve the realism of population-level descriptions.

A key modeling component is the functional response, which characterizes how predator consumption rates depend on prey density. Holling introduced several classes of functional responses to represent different feeding mechanisms [3]. Among these, the Holling type II

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functional response explicitly accounts for predator handling time and saturation effects, making it particularly suitable for modeling nonlinear predation processes. Consequently, this functional form has been widely adopted in studies addressing equilibrium stability, persistence, and oscillatory behavior in predator–prey systems [4], [5], [6].

Beyond predation, numerous biological factors have been incorporated into predator–prey models to better reflect empirical observations. These include migration [7], [8], refuge effects [9], disease dynamics [10], [11], and cannibalism [12]. Among these mechanisms, cannibalism, defined as the consumption of individuals of the same species, plays a significant role in population regulation and demographic structure [13], [14], [15]. Empirical evidence shows that cannibalism is widespread across taxa, including insects [16], amphibians [17], fish [18], mammals [19], and arthropods [20], indicating that it is a common ecological process rather than an exceptional phenomenon [21], [22].

In predator populations, cannibalism is frequently associated with stage structure due to size asymmetry and distinct energetic requirements between juvenile and adult individuals. Adult predators often prey upon juvenile conspecifics, generating stage-dependent cannibalistic interactions that affect recruitment, survival, and long-term population stability [23], [24]. Such interactions naturally motivate stage-structured modeling approaches, in which populations are divided into biologically meaningful classes to capture life stage-dependent processes.

Stage-structured cannibalism models have been studied in single-species settings, where interactions between juvenile and adult stages significantly influence population dynamics [25]. Extending this framework to predator-prey systems, Zhang et al. [26] proposed a three-dimensional model in which the predator population is structured into juvenile and adult classes, allowing cannibalism to emerge explicitly within the predator population. In their formulation, both predation and cannibalistic interactions are described using a Holling type I functional response, including maturation rate, and then the model exhibits rich stability and bifurcation behavior. In a complementary direction, Rayungsari et al. [27] investigated predator–prey dynamics with predator cannibalism and refuge effects within a two-dimensional framework using a Holling type II functional response, highlighting the importance of nonlinear predation in shaping coexistence and stability.

From a dynamical systems perspective, nonlinear predator–prey models often exhibit qualitative transitions in behavior as parameters vary, including the emergence of sustained population oscillations through Hopf bifurcation. Hopf bifurcation provides a rigorous mathematical mechanism for explaining recurrent population cycles, whereby the loss of stability of an equilibrium gives rise to periodic solutions. Such phenomena are well documented in predator–prey systems with nonlinear functional responses and are particularly pronounced in models incorporating cannibalism, where additional nonlinear feedback can destabilize equilibria and generate oscillatory dynamics [12], [28].

Motivated by these considerations, the present study formulates and analyzes a modified three–dimensional predator–prey model. This model explicitly incorporates predator stage structure and maturation rates[26], while both prey predation and cannibalistic interactions follow Holling type II functional responses[27]. The main objective is to investigate the qualitative dynamics of the model, with particular emphasis on equilibrium stability and Hopf bifurcation induced by biologically relevant parameters associated with adult predation intensity. By combining analytical techniques with numerical analysis, this work aims to clarify how stage-dependent cannibalism, maturation, and nonlinear predation jointly shape population dynamics.

The remainder of this paper is organized as follows. Section 2 describes the research methodology. Section 3 presents the results and discussion, covering the model formulation and its biological interpretation, existence and stability analysis, numerical simulations, and investigations of Hopf bifurcation. Finally, Section 4 provides the concluding remarks.

2 Methods

In analyzing the dynamics of the predator–prey system, several systematic steps are conducted to formulate and analyze the proposed model, which are described as follows.

1. Reviewing and examining relevant literature related to the problem is taken to establish a solid theoretical foundation for the model.
2. Formulating a predator–prey model that incorporates a Holling type II functional response, stage-structured, and predator cannibalism.
3. Identifying the existence and local stability of the equilibrium points in the model.
4. Investigating bifurcation phenomena in the model.
5. Performing numerical simulations to demonstrate and analyze the dynamical behavior of the model under various parameter values. Numerical simulations are performed with standard numerical integration methods for ordinary differential equations.

3 Results and Discussion

3.1 Mathematical Model

In this article, the mathematical model is formulated based on the following assumptions:

1. The prey population $x(t)$ grows logistically with intrinsic growth rate r and carrying capacity K, and is reduced through predation by adult predators following a Holling type II functional response, where m_1 denotes the maximum predation rate of adult predators on prey and n_1 represents the half-saturation constant measuring predator saturation at high prey density.
2. The predator population is divided into juvenile predators $y(t)$ and adult predators $z(t)$. Juvenile predators do not consume prey and are produced by adult predators at a constant per capita rate b.
3. Juvenile predators mature into adult predators at a constant per capita rate m, while adult predators prey upon juvenile predators through intraspecific cannibalism described by a Holling type II functional response, where m_2 denotes the maximum cannibalism rate and n_2 is the corresponding half-saturation constant.
4. The growth of the adult predator population results from the consumption of prey and juvenile predators with constant conversion efficiencies e_1 and e_2, respectively, whereas juvenile and adult predators experience natural mortality at rates d_1 and d_2.
5. The system evolves in a closed and homogeneous environment with no migration and constant environmental conditions.

From the assumptions stated above, the dynamics of the system are governed by the following set of ordinary differential equations:

$$\begin{aligned}
\frac{dx}{dt} &= rx\left(1-\frac{x}{K}\right)-\frac{m_1xz}{n_1+x}\\
\frac{dy}{dt} &= bz-\frac{m_2yz}{n_2+y}-my-d_1y \qquad (1)\\
\frac{dz}{dt} &= e_1\frac{m_1xz}{n_1+x}+e_2\frac{m_2yz}{n_2+y}+my-d_2z
\end{aligned}$$

where $x(t)$, $y(t)$, and $z(t)$ denote the densities of prey, juvenile predators, and adult predators at time t, respectively, with initial conditions $x(0)>0$, $y(0)>0$, and $z(0)>0$

3.2 Existence and Stability Analysis of Equilibrium Points

In this section, the equilibrium points of system (1) are obtained by solving

$$\begin{aligned} rx\left(1-\frac{x}{K}\right)-\frac{m_1xz}{n_1+x}&=0\\ bz-\frac{m_2yz}{n_2+y}-my-d_1y&=0\\ e_1\frac{m_1xz}{n_1+x}+e_2\frac{m_2yz}{n_2+y}+my-d_2z&=0 \end{aligned} \tag{2}$$

From system (2), we obtain the following equilibrium points, i.e:

1. The trivial equilibrium point
$$E_0=(0,0,0)$$
which always exists and corresponds to the extinction of all populations.
2. The prey-only equilibrium point
$$E_1=(K,0,0)$$
which always exists and represents the survival of the prey population in the absence of predators.
3. The coexistence equilibrium point
$$E_2=(x^*,y^*,z^*)$$
where $x^*>0$, $y^*>0$, and $z^*>0$ satisfy system (2). The coexistence equilibrium of system (1) is denoted by
$$E_3^*=(x^*,y^*,z^*)\in\mathbb{R}_+^3$$
From the prey equation, the adult predator density is given by
$$z^*=\frac{r(n_1+x^*)}{m_1}\left(1-\frac{x^*}{K}\right),\qquad 0<x^*<K \tag{3}$$
From the juvenile predator equation, y^* satisfies the quadratic equation
$$(m+d_1)y^2+\left[(m+d_1)n_2+(m_2-b)z^*\right]y-bn_2z^*=0 \tag{4}$$
which admits a unique positive solution
$$y^*=\frac{(b-m_2)z^*-(m+d_1)n_2+\sqrt{\Delta}}{2(m+d_1)} \tag{5}$$
where
$$\Delta=\left[(b-m_2)z^*-(m+d_1)n_2\right]^2+4(m+d_1)bn_2z^*$$
Substituting (3) and (5) into the predator equation yields a scalar equation
$$\Phi(x^*)=0 \tag{6}$$
which determines $x^*\in(0,K)$.
Consequently, the coexistence equilibrium can be written as
$$E_2^*=(x^*,\ y^*,\ z^*)$$

where x^* satisfies (6), and y^* and z^* are given by (5) and (3), respectively. The coexistence equilibrium E_2^* exists provided that the following conditions are satisfied:

$$\begin{aligned}
&\text{(H1) } r > 0, \quad K > 0\\
&\text{(H2) } b > m + d_1\\
&\text{(H3) } b > m_2\\
&\text{(H4) } \frac{rK}{n_1 + K} > \frac{m_1 d_2}{e_1 m_1 + e_2 m_2},\\
&\text{(H5) } d_2 < e_1 m_1 + e_2 m_2
\end{aligned}$$

Now, study the local stability of the dynamic system (1) around each of equilibrium points. The Jacobian matrix of the system (1) is determined as:

$$J(x,y,z) = \begin{pmatrix}
r\left(1 - \frac{2x}{K}\right) - \frac{m_1 z n_1}{(n_1 + x)^2} & 0 & -\frac{m_1 x}{n_1 + x}\\
0 & -\frac{m_2 z n_2}{(n_2 + y)^2} - (m + d_1) & b - \frac{m_2 y}{n_2 + y}\\
\frac{e_1 m_1 z n_1}{(n_1 + x)^2} & \frac{e_2 m_2 z n_2}{(n_2 + y)^2} + m & \frac{e_1 m_1 x}{n_1 + x} + \frac{e_2 m_2 y}{n_2 + y} - d_2
\end{pmatrix}$$

By evaluating the Jacobian matrix at each equilibrium point, the local stability properties of the equilibria are obtained as follows.

Theorem 1. *The trivial equilibrium point $E_0 = (0,0,0)$ is always unstable.*

Proof. The Jacobian matrix evaluated at E_0 is given by

$$J(E_0) = \begin{pmatrix} r & 0 & 0\\ 0 & -(m + d_1) & b\\ 0 & m & -d_2 \end{pmatrix}$$

The characteristic equation is

$$(\lambda - r)\left[\lambda^2 + (m + d_1 + d_2)\lambda + d_2(m + d_1) - bm\right] = 0$$

Thus, one eigenvalue is $\lambda_1 = r > 0$, while the remaining eigenvalues λ_2 and λ_3 are determined by the quadratic factor. Hence, the equilibrium point E_0 is unstable. □

Theorem 2. *If*

$$m + d_1 > 0 \quad \text{and} \quad d_2 > \frac{e_1 m_1 K}{n_1 + K}$$

then the prey-only equilibrium point E_1 of the system is locally asymptotically stable.

Proof. The Jacobian matrix evaluated in equilibrium point E_1 is given by

$$J(E_1) = \begin{pmatrix} -r & 0 & -\frac{m_1 K}{n_1 + K}\\ 0 & -(m + d_1) & b\\ 0 & 0 & \frac{e_1 m_1 K}{n_1 + K} - d_2 \end{pmatrix}$$

So, by solving the characteristic equation, we obtained the eigenvalues of $J(E_1)$ as

$$\lambda_1 = -r, \quad \lambda_2 = -(m + d_1), \quad \lambda_3 = \frac{e_1 m_1 K}{n_1 + K} - d_2$$

It means $\lambda_1 < 0$ and $\lambda_2 < 0$. Therefore, if

$$d_2 > \frac{e_1 m_1 K}{n_1 + K}$$

then $\lambda_3 < 0$, and each of the eigenvalues of $J(E_1)$ is negative. Hence, the equilibrium point E_1 is locally asymptotically stable. □

Theorem 3. *Let $E_2 = (x^*, y^*, z^*)$ be the non-trivial equilibrium point of the system. The equilibrium point E_2 is locally asymptotically stable if the coefficients a_1, a_2, and a_3 of the characteristic polynomial*

$$\lambda^3 + a_1\lambda^2 + a_2\lambda + a_3 = 0$$

satisfy the Routh–Hurwitz conditions

$$a_1 > 0, \qquad a_2 > 0, \qquad a_3 > 0, \quad \text{and} \quad a_1 a_2 > a_3$$

Proof. The Jacobian matrix of the system evaluated at the equilibrium point $E_2 = (x^*, y^*, z^*)$ is given by

$$J(E_2) = \begin{pmatrix} J_{11} & J_{12} & J_{13} \\ J_{21} & J_{22} & J_{23} \\ J_{31} & J_{32} & J_{33} \end{pmatrix}$$

where the entries are defined as follows:

$$J_{11} = r\left(1 - \frac{2x^*}{K}\right) - \frac{m_1 z^* n_1}{(n_1 + x^*)^2}$$

$$J_{12} = 0$$

$$J_{13} = -\frac{m_1 x^*}{n_1 + x^*}$$

$$J_{21} = 0$$

$$J_{22} = -\frac{m_2 z^* n_2}{(n_2 + y^*)^2} - (m + d_1)$$

$$J_{23} = b - \frac{m_2 y^*}{n_2 + y^*}$$

$$J_{31} = \frac{e_1 m_1 z^* n_1}{(n_1 + x^*)^2}$$

$$J_{32} = \frac{e_2 m_2 z^* n_2}{(n_2 + y^*)^2} + m$$

$$J_{33} = \frac{e_1 m_1 x^*}{n_1 + x^*} + \frac{e_2 m_2 y^*}{n_2 + y^*} - d_2$$

The characteristic equation of $J(E_2)$ is obtained from

$$\det(\lambda I - J(E_2)) = 0$$

which can be written in the form

$$\lambda^3 + a_1\lambda^2 + a_2\lambda + a_3 = 0$$

The coefficients a_1, a_2, and a_3 are given by

$$a_1 = -(J_{11} + J_{22} + J_{33})$$

$$a_2 = (J_{11}J_{22} - J_{12}J_{21}) + (J_{11}J_{33} - J_{13}J_{31}) + (J_{22}J_{33} - J_{23}J_{32})$$

$$a_3 = -\Big[J_{11}(J_{22}J_{33} - J_{23}J_{32}) - J_{12}(J_{21}J_{33} - J_{23}J_{31}) + J_{13}(J_{21}J_{32} - J_{22}J_{31})\Big]$$

According to the Routh–Hurwitz stability criterion, all roots of the characteristic equation have negative real parts if and only if

$$a_1 > 0, \qquad a_2 > 0, \qquad a_3 > 0, \quad \text{and} \quad a_1 a_2 > a_3$$

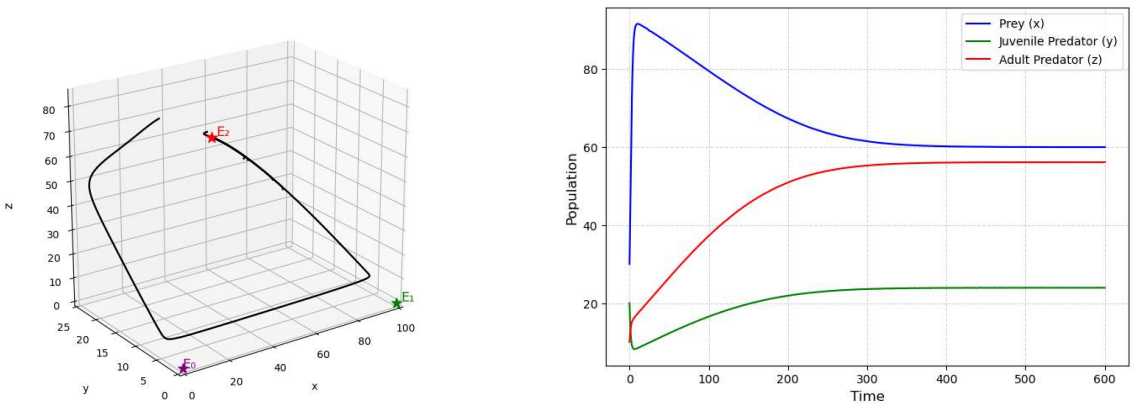
Therefore, under the above conditions, the non-trivial equilibrium point $E_2 = (x^*, y^*, z^*)$ is locally asymptotically stable. □

3.3 Numerical Simulation

Numerical simulations are carried out to illustrate the behavior of the solutions of model (1) near the equilibrium points, using parameter values that satisfy the theoretical conditions presented in Table 1.

Table 1: Parameter Value

Parameter	Description	Value	Reference
r	Intrinsic growth rate of prey	0.8	[29]
K	Carrying capacity of prey	100	[30], [31]
m_1	Maximum consumption rate of adult predators on prey	0.4	[27]
n_1	Half-saturation constant for prey consumption	10	assumption
m_2	Maximum cannibalism rate of adults on juvenile predators	0.2	assumption
n_2	Half-saturation constant for cannibalism	5	[32]
e_1	Conversion efficiency rate from prey to adult predators	0.2	[27], [33]
e_2	Conversion efficiency rate from juvenile predators to adults	0.15	[33]
b	Birth rate of juvenile predators produced by adults	0.4	[34]
m	Maturation rate from juvenile to adult predators	0.25	assumption
d_1	Natural mortality rate of juvenile predators	0.3	[33]
d_2	Natural mortality rate of adult predators	0.2	[33]



(a) Phase portrait showing convergence to E_2 **(b)** Time series evolution of the populations

Figure 1: Numerical simulations of the model for parameter set Table 1. (a) Trajectories contracting toward the stable equilibrium E_2, and (b) convergence of state variables to their respective steady states

We obtain the dynamic solution of system (1) under parameter set Table 1 to examine the numerical dynamics of the stage-structured predator–prey model with maturation and cannibalism. The numerical results depicted in Fig. 1 consist of a phase portrait and the corresponding time series. The phase portrait in Fig. 1a shows that the solution trajectories are attracted toward the positive coexistence equilibrium

$$E_2 = (59.9078,\ 23.9092,\ 56.0552),$$

indicating asymptotic convergence in the phase space. In addition, the time series in Fig. 1b illustrates the temporal evolution of the prey, juvenile predator, and adult predator populations,

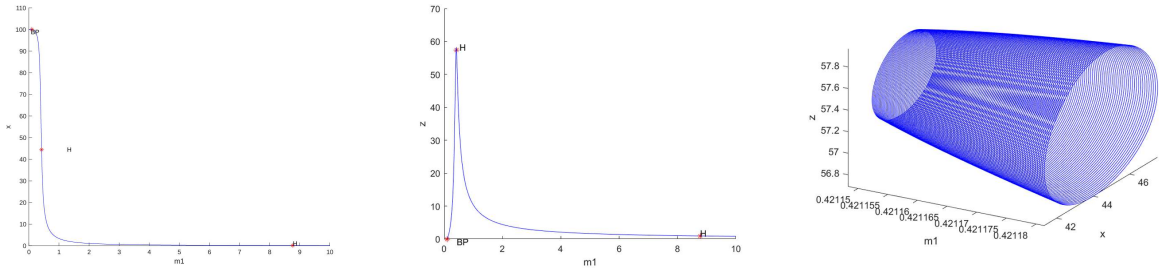
where all state variables exhibit decay of transient dynamics and approach constant steady state values.

The Jacobian operator associated with system (1), evaluated at the equilibrium point E_2, possesses a spectrum given by $\{\lambda_1, \lambda_2, \lambda_3\} = \{-0.1905, -0.0214, -0.7162\}$, which lies entirely in the open left half of the complex plane. By the spectral stability criterion for finite-dimensional dynamical systems, this spectral configuration implies that the coexistence equilibrium E_2 is locally asymptotically stable. Consequently, the qualitative behavior observed in both the phase portrait and the time series is in full agreement with the analytical stability results. These numerical simulations demonstrate that, for parameter set Table 1, the combined effects of stage structure, maturation, and cannibalism induce a stabilizing mechanism for system (1).

3.4 Hopf Bifurcation

In this section, numerical investigations of the Hopf bifurcation are performed using MATLAB and the continuation software MatCont. These simulations aim to validate the previous analytical findings regarding equilibrium stability and to illustrate the emergence of limit cycles. Specifically, we explore the influence of adult predation intensity on prey, maturation rates, and cannibalism on the system's transition into oscillatory dynamics.

3.4.1. *The Impact of Adult Predation Intensity on Oscillatory Dynamics*



(a) Time series of $x(t)$ for m_1 **(b)** Time series of $z(t)$ for m_1 **(c)** Hopf bifurcation manifold for m_1

Figure 2: Numerical bifurcation analysis for parameter m_1: (a), (b) population time series and (c) bifurcation manifold in (m_1, x, z)-space

We first investigate the dynamical response of system (1) to variations in the parameter m_1, which represents the maximum consumption rate of adult predators on prey and thus directly controls the strength of interspecific predation pressure. Numerical continuation of equilibria in the extended state parameter space (x, y, z, m_1) reveals two Hopf bifurcation points along the coexistence equilibrium branch, labeled H within the MatCont framework. The first Hopf point is located at

$$H_1 : \ (x, y, z, m_1) = (44.411994,\ 24.441966,\ 57.457628,\ 0.421132),$$

while the second Hopf point occurs at

$$H_2 : \ (x, y, z, m_1) = (0.144764,\ 0.634049,\ 0.923798,\ 8.772544).$$

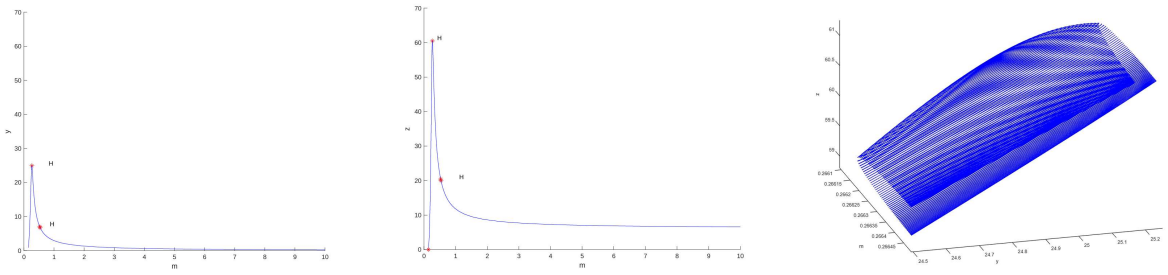
As m_1 crosses these critical values, the steady coexistence equilibrium loses stability and sustained oscillatory dynamics emerge through a Hopf bifurcation. This transition is clearly illustrated by the time series of the prey and adult predator populations shown in Fig. 2a and Fig. 2b, where stable equilibria give way to persistent periodic oscillations. The corresponding three-dimensional projection in the (m_1, x, z) space displayed in Fig. 2c highlights the geometric organization of the equilibrium branch and the associated stability changes induced by variations in adult predation intensity. In particular, the continuation of periodic solutions emanating from H_1 illustrates how the amplitude and phase structure of oscillations evolve smoothly as m_1 varies.

In addition, a branch point labeled BP is detected at

$$BP: \ (x, y, z, m_1) = (100.00, \ 0.00, \ 0.00, \ 0.10),$$

as indicated in Fig. 2a. This point corresponds to a degenerate prey-only equilibrium at which the coexistence equilibrium branch intersects the trivial solution manifold, acting as an organizing center for the global bifurcation structure. The coexistence of two Hopf bifurcation points along the same equilibrium continuation curve, together with the branch point BP, highlights the strong sensitivity of system (1) to variations in the adult predation parameter m_1. These results demonstrate that increasing the maximum consumption rate of adult predators can generate multiple oscillatory regimes, underscoring the central role of interspecific predation in shaping the nonlinear dynamics of system (1).

3.4.2. *The Impact of Maturation Rate on Stage-Structured Oscillatory Dynamics*



(a) Time series of $y(t)$ for m **(b)** Time series of $z(t)$ for m **(c)** Hopf bifurcation manifold for m

Figure 3: Numerical bifurcation analysis for parameter m: (a), (b) population time series and (c) bifurcation manifold in (m, y, z)-space

We next analyze system (1) with respect to the maturation parameter m, which governs the transition rate from juvenile to adult predators and therefore controls the internal population flux within the predator population. Numerical continuation of equilibria in the extended state parameter space (x, y, z, m) reveals two Hopf bifurcation points along the coexistence equilibrium branch, denoted by H. The first Hopf point is detected at

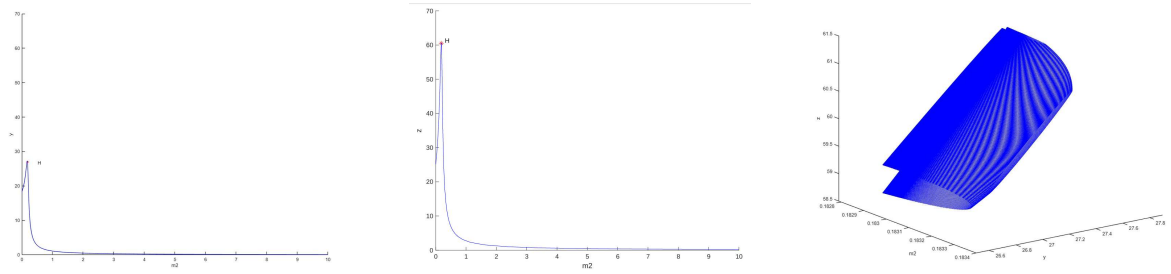
$$H_1: \ (x, y, z, m) = (44.398902, \ 24.939956, \ 60.492774, \ 0.266121),$$

while the second Hopf point occurs at

$$H_2: \ (x, y, z, m) = (0.250470, \ 7.004695, \ 20.449591, \ 0.527072).$$

Crossing either of these critical values leads to a loss of stability of the coexistence equilibrium and the emergence of sustained periodic oscillations. This behavior is confirmed by the time series of the juvenile and adult predator populations shown in Fig. 3a and Fig. 3b, where the transition from steady-state dynamics to recurrent oscillations is clearly observed. The three-dimensional phase portrait in the (y, z, m) space displayed in Fig. 3c illustrates the continuation of periodic solutions originating from H_1, forming a curved half-loop-like surface. This geometric structure should not be interpreted as an incomplete bifurcation; rather, it represents the solution manifold of limit cycles parameterized by m. For each fixed value of m, the dynamics in the (y, z) phase plane consist of a closed periodic orbit, while their embedding into the (y, z, m) space captures the systematic variation of oscillation amplitude and phase induced by changes in the maturation rate. Biologically, these results highlight that maturation acts as a key internal timescale capable of destabilizing steady coexistence and organizing oscillatory dynamics in stage-structured predator–prey interactions.

3.4.3. *The Impact of Cannibalism Intensity on Hopf Bifurcation and Periodic Dynamics*



(a) Time series of $y(t)$ for m_2 **(b)** Time series of $z(t)$ for m_2 **(c)** Hopf bifurcation manifold for m_2

Figure 4: Numerical bifurcation analysis for parameter m_2: (a), (b) population time series and (c) bifurcation manifold in (m_2, y, z)-space

Finally, we examine the role of the cannibalism parameter m_2, which represents the maximum cannibalism rate of adult predators on juvenile predators and thus quantifies the intensity of intraspecific predation within the predator population. Numerical continuation of equilibria in the extended state parameter space (x, y, z, m_2) reveals a Hopf bifurcation point along the coexistence equilibrium branch, labeled H, located at

$$H: \ (x, y, z, m_2) = (44.478173,\ 26.973268,\ 60.494554,\ 0.183455).$$

As m_2 crosses this critical value, the coexistence equilibrium loses stability and sustained oscillatory dynamics emerge, indicating that sufficiently strong cannibalistic pressure can destabilize population balance and induce persistent oscillations. This behavior is confirmed by the time series population shown in Fig. 4a and Fig. 4b. The continuation of periodic solutions originating from H manifests as a curved, half-loop-like surface in the (y, z, m_2) space, as shown in Fig. 4c. This geometry should not be misinterpreted as an incomplete limit cycle or a numerical artifact; rather, it reflects the biologically admissible branch of periodic solutions when m_2 is treated as a bifurcation parameter. For each fixed value of m_2, the dynamics in the (y, z) phase plane consist of a closed periodic orbit, while the projection into the (y, z, m_2) space captures how the amplitude and position of these oscillations evolve with increasing cannibalism intensity. These results demonstrate that cannibalism introduces a strong nonlinear intraspecific feedback mechanism, capable of generating and organizing oscillatory dynamics through Hopf bifurcation in the stage-structured predator–prey system.

4 Conclusion

This study examined a three-dimensional predator–prey model with Holling type II functional responses, explicitly incorporating predator stage structure and cannibalism. The primary objective was to determine how biologically meaningful parameters governing adult predation, maturation, and cannibalism influence equilibrium stability and the onset of oscillatory dynamics through Hopf bifurcation. For the parameter set listed in Table Table 1, numerical simulations show that all trajectories converge asymptotically to the positive coexistence equilibrium

$$E_2 = (59.9078,\ 23.9092,\ 56.0552).$$

This observation is rigorously supported by the spectrum of the Jacobian matrix evaluated at E_2, given by $\{-0.1905, -0.0214, -0.7162\}$, which guarantees local asymptotic stability since all eigenvalues lie in the open left half–plane.

Numerical continuation reveals that this stable regime is bounded by multiple Hopf bifurcation thresholds associated with distinct biological mechanisms. Specifically, continuation with respect to the adult predation parameter m_1 identifies two Hopf bifurcation points at $m_1 \approx 0.4211$ and

$m_1 \approx 8.7725$, while variation of the maturation rate m yields Hopf bifurcations at $m \approx 0.2661$ and $m \approx 0.5271$. These results indicate that, although the system remains locally asymptotically stable at the simulation values $m_1 = 0.4$ and $m = 0.25$, it operates in close proximity to critical thresholds where internal stage-transition dynamics can trigger sustained oscillatory behavior.

Furthermore, analysis of the cannibalism parameter m_2 identifies a Hopf bifurcation at $m_2 \approx 0.1835$. Numerical continuation of the resulting periodic solutions reveals a nontrivial solution manifold, characterized by a curved, half-loop–like geometry in the (y, z, m_2) space. This geometric structure reflects the biologically admissible branch of periodic orbits and illustrates that, while cannibalism introduces a destabilizing nonlinear feedback, the persistence and amplitude of population cycles are regulated through its interaction with predator maturation.

Overall, the results demonstrate that oscillatory dynamics in the proposed stage-structured predator–prey model arises through multiple, quantitatively distinct Hopf bifurcation mechanisms. Adult predation, maturation, and cannibalism define separate instability boundaries, and their interplay governs the transition from stable coexistence to periodic population cycles. Although the present study relies on numerical bifurcation analysis, it provides a mathematically consistent foundation for future work aimed at deriving analytical Hopf conditions and extending the model to incorporate additional ecological mechanisms such as spatial diffusion or time delays.

CRediT Authorship Contribution Statement

Sela Tri Indah Sari: Conceptualization, Methodology, Software, Formal Analysis, Investigation, Writing–Original Draft, Visualization. **Dian Savitri:** Conceptualization, Methodology, Validation, Writing–Review & Editing, Supervision.

Declaration of Generative AI and AI-assisted technologies

During the preparation of this study, the authors additionally utilized ChatGPT to help with structural refinement and preliminary drafting, while Grammarly was used to improve grammatical accuracy and sentence structure. The writers accept full responsibility for the publication's content and have reviewed and amended it as necessary.

Declaration of Competing Interest

The authors declare no competing interests.

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Data and Code Availability

The data and code that support the findings of this study are available in public repositories and can be accessed openly.

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